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An alternative proposal for interpreting two recurring misinterpretations in the relativistic mass-energy relationship

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Abstract

This essay explores two recurring mistakes commonly encountered in the teaching of the mass-energy relationship within the framework of the Special Theory of Relativity. Firstly, there are interpretations positing the possibility of converting mass into energy (and vice versa), and secondly, interpretations suggesting an equivalence or sameness between mass and energy. These conceptual misunderstandings are systematically dismantled through a meticulous historical-philosophical and conceptual analysis. Consequently, the aim is to empower educators and university professors with the means to preempt the perpetuation of these misguided notions. We advocate for an interpretive approach grounded in scholarly literature, drawing upon the seminal contributions of Warren, Martins, and Miller. Their scholarly endeavors furnish invaluable insights into the intricate fabric of the mass-energy relationship within the confines of the Special Theory of Relativity, underscored by the imperative of a nuanced and contextual comprehension of this foundational concept.

Keywords: Theory of Special Relativity, Teaching Physics, Misconceptions.

I. INTRODUCTION

This essay is intended to be a contribution to the teaching of relativity for both elementary and university education. Our focus is on the mass-energy relationship and two recurrent and persistent misconceptions associated with it, namely: the idea that Einstein was the first to discover it in 1905; that the relationship establishes that mass can be transformed into energy (and vice versa); that the relationship established that mass and energy are equivalent (or the same thing). These are recurrent and persistent, as they appear in several works (French, 1968; Resnick, 1968; Thornton, Rex, Hood, 2021; Knight, 2023) as reported by Warren (1976), Nunes & Queirós (2020) and Nunes, Queirós & Cunha (2022).

However, even experts are subject to making and reproducing mistakes, and that is why, in this essay, we propose to deconstruct these two mistakes. In section 2, we will briefly discuss the history of the mass-energy relationship in relativity theory. For this, we undertake historical research in the historiography of special relativity. In section 3, we deconstruct the two conceptual misconceptions about the meaning of the mass-energy relationship. For that, we used a semantic analysis of the equation and its interpretations. Finally, in section 4, we make a brief reflection on what has been exposed.

To make the text more accessible, we have omitted mathematical developments and opted for a qualitative description. In this way, even the reader with some difficulty in elementary mathematics can benefit from this text. Finally, we hope that this essay reaches educators and university professors and helps to eradicate these old, recurrent and persistent errors about the mass-energy relationship.

II. HISTORICAL NOTES ON THE MASS-ENERGY RATIO

A widespread story in popular science and in relativity manuals is that in 1905, Albert Einstein (1905) discovered that mass and energy are associated by a simple mathematical relationship: $E = mc^2$. Although the mass-energy relationship has become a symbol of the Special Theory of Relativity and Albert Einstein, the emergence of this equation predates both Relativity and Einstein (Ives, 1952, Martins, 1989, 2012, 2015, Fadner, 2008). The mass-energy relationship emerged as a result of studies on the dynamics of the electron. Later, this research tradition was incorporated into the recently appeared Theory of Special Relativity, based on the work of Poincaré, Einstein, Minkowski and Planck (Martins, 2015).

To better understand the relationship in question, we will introduce two physical examples. The first, proposed by Poincaré (1900), suggests that radiation possesses inertia. Thus, when we attempt to deflect a beam of light, it will exhibit a resistance expressed by the relation $M=E/c^2$. In this way, it is possible to associate a mass with light, known as maupertuisian mass or Poincaré mass (Martins, 1989, 2012).

The second example is drawn from Einstein's original essay of 1905. According to the German physicist, the energy content also contributes to the total inertia of a system. Therefore, when a radioactive salt emits radiation (energy), its inertia decreases in proportion to $M=E/c^2$. This implies that whenever the energy of a closed system varies, its inertia also varies. For this reason, the Sun loses mass as it emits radiation into outer space. Additionally, a heated body exhibits greater inertia compared to its cooled state.

In a global sense, the mass-energy relationship applies to closed systems that are not subject to external pressures, allowing for the consideration of the total mass and total energy of a system as interconnected, regardless of its specific location (Martins, 2012). This means that in an isolated system, a change in energy will result in a corresponding change in mass, even if the system is in motion or in different positions in space.

Furthermore, the invariance of the mass-energy relationship is another fundamental characteristic that makes it essential in physics. Rest mass is an invariant quantity, associated with the norm of the 4-momentum, while energy, which corresponds to the time component of the 4-momentum, depends on the reference frame because it involves both rest mass and the object's linear momentum. The famous equation $E=mc^2$ is valid only in the center-of-mass frame, where linear momentum is zero, and energy is exclusively determined by the rest mass. In other reference frames, energy includes the contribution of motion, making it a quantity that varies depending on the observer.

In a historical perspective, the first record of this equation occurred in 1900, in a work by Poincaré (1900), entitled The Reaction Principle in Lorentz Theory (*cf* Martins, 1989, 2012, 2015, Fadner, 2008). At that time, it seemed that Lorentz electrodynamics was irreconcilable with the Principle of Action and Reaction (and therefore with the Conservation of Linear Momentum). Lorentz suggested that the new mechanics should give up this principle (Poincaré, 1908), however, Poincaré believed that it would be possible to make a compromise. Thus, in 1900, Poincaré, taking the principle of relativity as a starting point, showed that if radiation behaved like a subtle fluid, with a center of mass and whose inertia was measured by the ratio of its energy and the speed of light squared *c*, then it would be possible to preserve the reaction principle in Lorentz theory. Later, it seems that Poincaré abandoned this conception in favor of rejecting the principle of conservation of momentum (Poincaré, 1908).

In 1904, F. Hasenöhrl, based on M. Abraham's studies on the radiation pressure in moving mirrors, showed that a box full of radiation presented greater resistance to changes in movement if this box was without radiation (Martins, 1989, 2012, 2015, Fadner, 2008). Hasenöhrl also established the relationship between additional inertia and radiation energy, however, he made a calculation error and reached the wrong value (Martins, 2015). This error was corrected, in 1905, by Abraham, and recognized by Hasenöhrl himself: the additional inertia was $4/3\ E/c^2$ (Ives, 1952, Fadner, 2008, Martins, 2012, 2015). In that same year, Poincaré (1905, 1906) showed that the equilibrium of the electron suffering external distensions, as in the case of the box full of light, required the existence of negative pressures that resulted in a reduction of inertia by a factor of $1/3$. Thus, if a box were filled with radiation, there would be an increase of $4/3\ E/c^2$ and a decrease of $1/3\ E/c^2$, due to the Poincaré pressures, so the net increase in inertia would be E/c^2 .

In 1905, after the publication of the works of Abraham, Hasenöhrl and Poincaré, Einstein "showed" that this equation was a consequence of the Principle of Relativity (Einstein, 1905). We put the word "showed" in quotation marks because in 1907, Planck challenged Einstein's deduction, showing that it was valid only as a first-order approximation (Planck, 1907; *cf.* Ives, 1952). However, the most serious objection was presented by Ives (1952) who argued that Einstein begged the question and therefore the deduction was not valid. In this work, Ives argues that the valid deductions are those proposed by Poincaré and Hasenhörl (Ives, 1952). In defense of Einstein, Stachel and Torretti showed that Einstein's deduction is locally valid (Stachel, Torretti, 1982).

In addition to the syntactic issues discussed by Planck (1907), Ives (1952), and Stachel & Torretti (1982), such as Einstein's potential petitio principii—where he posits the energetic terms that he later seeks to deduce—there are also semantic difficulties. These include the difficulty of establishing energy and mass as equivalent or as manifestations of the same substance. While Poincaré, Hasenöhrl and Abraham studied singular cases and limited themselves to making inferences about them, Einstein started from a singular case and stated that its consequences were universal (Martins, 2015). It can be said that Einstein took a "logical leap" and the consequences were perceived by Planck who showed that this equation is not a general law (Planck, 1907). For extensive systems subjected to pressure, a new relationship should be used: the mass-enthalpy relationship (Martins, 2015). We also know that this relation does not apply to electric potential energy (Martins, 1989, 2012, 2015).

These controversies are recurrent in science and show that the progress of science is not linear and that theories do not emerge "ready-made" and immune to revision. Discussing these points in physics and relativity classes is to show the critical and polemical dimension that characterizes good science.

After these historical considerations, we will point out two misconceptions that have become common and persistent involving the mass-energy relationship.

III. CONCEPTUAL MISTAKES INVOLVING THE MASS-ENERGY RELATIONSHIP

In the previous section, we saw a brief outline of the genesis and development of the mass-energy relationship. Despite the controversies about its validity limits and the forms of derivation, its principle is quite simple: "*certain forms of energy present an inertia given by* $m= E/c^2$" and, reciprocally, "*for a closed system, in the absence of voltages or external pressures, its inertia is a measure of its energy, given by* $E = mc^2$" (Einstein, 1905; Warren, 1976; Martins, 2012).

However, for some mysterious reason, this equation originated at least two misinterpretations (Warren, 1976), namely: that it is possible to transform mass into energy (and vice versa) & that mass and energy are equivalent (or identical). As we will see below, none of these interpretations are correct and university educators and professors should not only avoid them, but also warn their students and academics about these mistakes.

A. Conversion of Mass into Energy and vice versa

In addition to these inadequacies, other semantic problems arose: the idea spread that this equation establishes a transformation of mass (or, matter) into energy (and vice versa) or an equivalence between mass and energy. According to Warren this is a common misconception (Warren, 1976). Both the syntax of the equation and the grammar of the theory of relativity do not authorize deriving such conclusions.

> *This idea is absolutely contrary to Einstein's principle. According to Einstein if we start with a mass M this has energy content* $E = Mc^2$*. If this energy is changed to any other form there is still the same quantity E which still has the same mass M. Whilst the mass-energy relation explicitly depends upon the conservation of energy and establishes the Conservation of mass, the idea of 'conversion' certainly implies non-conservation of both. Naturally students who read about conversion and do not know the principles of the theory believe that the mass-energy relation is contrary to the conservation laws. What is surprising is that they are taught such a wrong idea in the first place.* (Warren, 1976, p. 53)

Furthermore, if such a conclusion were syntactically and semantically possible, we could infer that other relations, such as Planck's Law, also establish the interchange or equivalence between different magnitudes.

> *It is worth comparing the equation* $E = mc^2$ *from relativity with the relation* $E = hf$ *from quantum theory. Why, in the second case, does nobody talk about equivalence between energy and frequency, or transformation of energy into frequency? In both cases we have a universal constant (c or h) relating two physical quantities, which are proportional. These two equations are exactly of the same nature. None of them establishes either equivalence or the possibility of transforming one thing into another.* (Martins, 2012, p. 128)

Therefore, the way that seems most appropriate to us for reading this equation is: "some forms of energy have an inertia given by E/c^2". In this way, the restricted character is recovered and inferences about the interchange and equivalence between mass and energy are avoided.

In summary, based on this brief exposition, we argue that the approaches to the mass and energy relationship are unsatisfactory, from a historical point of view, when they omit information about its development and are restricted only to the work of Albert Einstein, and will be inadequate, from the conceptual point of view, when they affirm the possibility of transforming energy into mass and vice versa. Unfortunately, occurrences of these errors in the literature and in books that address the subject are not rare (Nunes, Queirós, 2020; Nunes, Queirós, Cunha, 2022)

Unfortunately, the work by Knight (2023, p. 1107) exemplifies this mistake. The passage below clearly highlights this error: "Mass and energy (...) are equivalent in the sense that mass can be transformed into energy and energy can be transformed into mass as long as the total energy is conserved."

Next, we will address another misinterpretation of this relationship, which in our brief research proved to be more frequent and more persistent: the equivalence between mass and energy.

B. Mass-Energy Equivalence

A widespread conception is that the mass-energy relationship establishes an equivalence between these two magnitudes. We find this statement both in popular science books and in relativity manuals (*cf*. French, 1968; Resnick, 1968; Thornton, Rex, Hood, 2021; Knight, 2023). However, Martins disputes this interpretation. Let's analyze his argument:

> *According to the very meaning of the word equivalent, two things can be considered equivalent when they "have the same value", in some sense. For example, 32° Fahrenheit is equivalent to 0° Celsius and 4.2 Joules is equivalent to 1 calorie. In these two examples, we are using different names to indicate the same thing (a certain temperature, or a certain amount of energy). It's like using the names "morning star" and "evening star" to indicate Venus: they are different designations for the same object. If mass and energy were equivalent, it would mean that these words represent the same thing, described in different ways. In that case, the relation $E = mc^2$ would be a simple kind of definition and would not be a physical law. No one considers, for example, that the equation for converting calories to joules or from Celsius temperature to Fahrenheit temperature is a physical law. If $E = mc^2$ established an equivalence between mass and energy, this relationship could never predict phenomena or be experimentally tested, and its scientific utility would be very small.* (Martins, 2012, p. 125)

To define equivalence, Martins resorts to the concept of extension of formal logic and philosophy of language. In these disciplines (and their correlates) we say that two expressions are equivalent if they have the same extension (Miller, 2007). For example: “morning star” and “evening star” are equivalent expressions, because both have the same extension, namely the planet Venus. The Portuguese and English words, respectively, “*Mesa*” and “*Table*” are equivalent, as they refer to the same object.

In the case of different measures, we will say that a necessary condition for them to be equivalent is that they refer to the same quantity (Gibbings, 2011). The classic example is the equivalence between heat and energy, 1 cal = 4.184J. Within dimensional analysis, we can recognize fundamental quantities, such as Mass, and quantities derived from fundamentals, such as Energy (Gibbings, 2011).

On the other hand, we can grant that the choice of what is fundamental and what is derived is conventional, what must be preserved are the relations between the magnitudes. Thus, we could build an equally coherent system where Energy, for example, is the fundamental quantity. From this information, we can assess whether it is logically possible to state that Mass and Energy are equivalent.

For there to be equivalence between Inertia and Energy, it is necessary that they refer to the same Magnitude. But while Mass is a fundamental quantity, Energy is a derived quantity. Therefore, they are different magnitudes, therefore, with different extensions, in this way they cannot be equivalent. However, we could look for a system where Mass and Energy are two fundamental quantities and preserve the relationships between the quantities, so that we can then establish an equivalence between them. We will prove by reductio ad absurdum that this is impossible.

We will assume as true the hypothesis: there is a system where mass and energy are fundamental quantities. As in this system, the mass-energy relationship, $E=mc^2$, must be preserved, we must ask ourselves, is c a fundamental or derived physical quantity? If it is fundamental, then either m or E is derivative, but our assumption is that both are fundamental, so c is a derived quantity. However, if c is a derived quantity, it must be composed of fundamental quantities, which implies that either m or E must also be derivatives to maintain the dimension of meters per second ($[L][T]^{-1}$). If both m and E are fundamental quantities, we would have the velocity expressed as the square root of units of mass divided by energy. This would imply that mass is the square of space and energy is the square of time. Therefore, they would not be fundamental quantities but derived ones. Thus, we arrive at an absurdity. Therefore, our hypothesis is false. Thus, there is no system where mass and energy are fundamental quantities. If Mass and Energy cannot simultaneously be fundamental magnitudes, then they cannot have the same extension and, therefore, there is no equivalence between Mass and Energy.

Henceforth, it behooves us to concur with Martins (2012) and deduce that the nexus between mass and energy does not presage an equivalence betwixt these magnitudes. Ergo, treatises purporting an equivalence or indeed an identity betwixt mass and energy are to be scrutinized with circumspection. Illustratively,

1. Resnick's (1968, p. 31) very section 3.6 of Chapter 3 bears the appellative "The Equivalence of Mass and Energy.
2. French (1968, p. 17) endeavors to substantiate such equivalence by invoking the following exemplar: " The prime example of the mass-energy equivalence, to which we owe our continuing existence, is provided by thermonuclear

reactions occurring in stars such as the sun" (later on, we shall elucidate the accurate interpretation of this phenomenon).

3. Thornton, Rex, and Hood (2021, p. 67), akin to Resnick (1968), proffer a subsection within section 2.12, entitled "Equivalence of Mass and Energy," wherein they too endeavor to rationalize said phenomenon.

Indeed, Knight (2023), akin to French (1968), although acknowledging that mass and energy are distinct magnitudes, persists in their equivalence, as evidenced in the already mentioned excerpt: "Mass and energy are not the same thing, but, as the last few examples have shown, they are equivalent in the sense that mass can be transformed into energy and energy can be transformed into mass as long as the total energy is conserved." (Knight, 2023, p. 1107).

IV. A PROPOSAL FOR INTERPRETING THE RELATIVISTIC MASS-ENERGY RELATIONSHIP

In order to clarify the meaning of the mass-energy relationship, we must initially introduce the conditions under which this equation can be taken as pragmatically true (quasi-true) (*cf.* Mikenberg, Da Costa, Chuaqui, 1986; Bueno, De Souza, 1996; Da Costa, 1989, 2018.; Da Costa, French, 2003; Hifume, 2003; DàOttaviano, Hifume, 2007).

[C_{EST}]: Einstein-Stachel-Torreti Condition: The premisses to derive the mass—energy equivalence relation are: “the principle of relativity, the law of conservation of energy, the existence of a Newtonian limit for relativistic dynamics, and the relativistic law of transformation of the energy of an electromagnetic wave.” (Stachel, Torreti, 1982, p. 762).

For any physical system that satisfies [C_{EST}], we can present two valid interpretations for the equation $E = mc^2$.

[I_1] Interpretation of Poincaré-Langevin: *Some forms of Energy, whose magnitude is E, have an inertia m, whose magnitude is given by $m = E/c^2$* (*cf.* Poincaré, 1900; Langevin, 1913; Ives, 1952; Martins, 2012, 2015).

[I_2] Interpretation of Einstein-Martins: “*If the total energy of an isolated system is E, then this system has an inertial mass $m = E/c^2$*” (Martins, 2012, 2015; *cf.* Einstein, 1905).

Thus, [C_{EST}] determines under which circumstances the mass-energy relationship is valid (thus, it is not a Universal Law), while [I_1] and [I_2], provide the proper readings of this relationship, in addition to imposing two new restrictions, namely: (i) the relationship does not apply to all forms of energy; (ii) the relationship only applies to isolated systems.

For example, suppose that in a nuclear fission process, there is a 0.10% reduction in the inertia of the fissionable material. Instead of saying that "after fission, 0.10% of the mass was converted into released energy", we should say that "the released energy has an inertia equal to 0.10% of the mass of the material before fission". In this way, it is clear that the reduction of inertia is not due to the conversion of mass into energy, but because energy presents an inertia. This is even Einstein's (1905) original interpretation in his 1905 essay.

Finally, it is worth adding that in the discussion of invariance, it is important to clarify that the mass-energy relationship generally refers to rest mass and total energy in a specific reference frame, such as the center of mass. The relationship is valid for closed systems, where the conservation of mass and energy can be observed. The quantities mentioned are generally local, but can be applied to global systems as long as the interactions are adequately considered.

V. CONCLUSIONS

Throughout this article, we show two recurring misconceptions in popular science texts and relativity manuals over the years. Our intention is not to demean these works and invalidate all their content, but to alert educators and university professors that it is necessary to maintain a vigilant and critical attitude. They not only can, but should continue to be used in the classroom, but always with a critical reading, making observations and raising possible divergences. In other words, it's about bringing the controversy that characterize good science into the classroom.

REFERENCES

Bueno, O., & De Souza, E. G. (1996). The concept of quasi-truth. *Logique et Analyse*, 39(153/154), 183-199.

Da Costa, N. C. A. (1989). Logic and pragmatic truth. *Studies in Logic and the Foundations of Mathematics*, 126, 247-261.

Da Costa, N. C. A. (2018). *O conhecimento científico*. São Paulo: Paulus.

Da Costa, N. C. A., & French, S. (2003). *Science and partial truth: A unitary approach to models and scientific reasoning*. Oxford: Oxford University Press.

D'à Ottaviano, I. M. L., & Hifume, C. (2007). Peircean pragmatic truth and da Costa's quasi-truth. In L. Magnani & P. Li (Eds.), *Model-based reasoning in science, technology, and medicine* (Vol. 64, pp. 553-591). Berlin, Heidelberg: Springer.

Einstein, A. (1905). Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig? *Annalen der Physik*, 323(13), 639-641.

Einstein, A. (1907). Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen. *Jahrbuch der Radioaktivität und Elektronik*, 4, 411-462.

Fadner, W. L. (2008). Did Einstein really discover ''E=mc²''? *American Journal of Physics*, 56(2), 114-122.

French, A. P. (1968). *Special relativity*. New York: W. W. Norton & Company, Inc.

Gibbings, J. C. (2011). *Dimensional analysis*. London: Springer.

Hifume, C. (2003). *A pragmatic theory of truth: Newton C.A. da Costa's quasi-truth*. Campinas: State University of Campinas.

Ives, H. (1952). Derivation of the mass-energy relation. *Journal of the Optical Society of America*, 42(8), 540-543.

Knight, R. D. (2023). Relativity. In *Physics for scientists and engineers: A strategic approach* (5th ed.). San Francisco: Pearson Addison-Wesley.

Langevin, P. (1913). L'inertie de l'énergie et ses conséquences. *Journal de Physique Théorique et Appliquée*, 3, 553-591.

Martins, R. A. (1989). A relação massa-energia e energia potencial. *Caderno Catarinense de Ensino de Física*, 15, 265-300.

Martins, R. A. (2012). *Teoria da relatividade especial*. Livraria da Física.

Martins, R. A. (2015). *A origem histórica da relatividade especial*. Livraria da Física.

Mikenberg, I., Da Costa, N. C. A., & Chuaqui, R. (1986). Pragmatic truth and approximation to truth. *The Journal of Symbolic Logic*, 51(1), 201-221.

Miller, A. (2007). *Philosophy of language* (2nd ed.). Montreal: McGill-Queen's University Press.

Nunes, R. C., & Queirós, W. P. (2020). Doze mitos sobre a teoria da relatividade que precisamos superar. *Caderno Brasileiro de Ensino de Física*, 37(2), 531-573.

Nunes, R. C., Queirós, W. P., & Cunha, J. A. R. (2022). Conceito de massa e a relação massa-energia no conteúdo de relatividade especial em livros didáticos de física. *Revista de Enseñanza de la Física*, 34(1), 8-20.

Planck, M. (1907). Zur Dynamik bewegter Systeme. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin*, 29, 542-570.

Poincaré, H. (1900). La théorie de Lorentz et le principe de réaction. *Archives néerlandaises des sciences exactes et naturelles*, 5, 252-278.

Poincaré, H. (1908). La dynamique de l'électron. *Revue générale des sciences pures et appliquées*, 19, 386-402.

Resnick, R. (1968). *Introduction to special relativity*. New York: John Wiley & Sons, Inc.

Stachel, J., & Torretti, R. (1982). Einstein's first derivation of mass–energy equivalence. *American Journal of Physics*, 50(2), 760-763.

Thornton, S. T., & Rex, A. (2021). *Modern physics for scientists and engineers* (5th ed.). Boston: Cengage.

Warren, J. W. (1976). The mystery of mass-energy. *Physics Education*, 11(1), 52-54.