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Kantian Transcendental Idealism in Physics Teaching: The Universe on a Sheet of Paper

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Abstract

One of the most attractive aspects of Theoretical Physics is that its implications are often more fascinating than fiction. Both Philosophy and Physics often use logical reasoning that involves experiments that cannot be carried out in practice, but that through imagination allow us to explore and understand non-experimental aspects of the Universe. Einstein popularized the term "thought experiment" for conceptual approaches explored only in the field of ideas. In this work, we discuss how knowledge is possible through reflections that lead the subject from a simple logic to a transcendental logic from Kantian Transcendental Idealism. Subsequently, we present an initially simple experiment to be carried out in practice, involving folding a sheet of paper. Then, after verifying the existence of an experimental limit for its continuity, we performed it through a theoretical-mental extension and proceeded in a guided way to explore the Universe.

Keywords: Kantian Transcendental Idealism; Thought Experiments; Scientific Dissemination.

I. INTRODUCTION

"*I could be bounded in a nutshell and count myself a king of infinite space*" (Shakespeare, 2015). It was this line from Hamlet, Act 2, Scene 2, by William Shakespeare, that inspired the title of the book "The Universe in a Nutshell" by Stephen Hawking (2001). Hamlet may have teased us by saying that although we human beings are physically limited, we have our minds free to explore the Universe, and perhaps even a universe of Universes. Hawking's (successful) attempt to explore our Universe and its laws for different audiences, by stimulating imagination and creativity, inspired a legion of readers to be interested in science, through scientific dissemination.

Many of these readers even became scientists, and probably many physicists. Questions like: "How big is the planet compared to humans? And the Universe? And how small is an atom? These are some examples of questions raised at the dawn of science, at the dawn of each revolution in collective and individual scientific thinking. Scientific popularizers such as Carl Sagan and many others have focused and continue to focus on the field of formal and non-formal scientific language, which basically consists of promoting profoundly abstract discussions and reflections on Science and Philosophy for the most diverse audiences through an accessible language to each one of them, in a way that creates cognitive conditions for knowledge (Sagan, 2006, 2017, 2019). These cognitive conditions were the object of investigation by the German philosopher Immanuel Kant, who proposed that it is possible to explore the cognitions intrinsic to human beings, in order to favor the understanding of the world around them.

Kant (1998) proposes that, through sensations and simple logic, the subject passes to a Transcendental Logic, going beyond common limits and entering the world of ideas. This Kantian reasoning intersects reflections raised by Paulo Freire (Freire, 2017, 2019, 2020; Freire and Macedo, 2021). For these two thinkers, appreciation occurs as understanding occurs. In this sense, there is a causal relationship between understanding the real world and understanding the abstract world. In this work, we discuss Kantian Transcendental Idealism in Physics Teaching and present a proposal for a thought experiment that explores the idea of a transcendent subject.

II. KANTIAN TRANSCENDENTAL IDEALISM IN PHYSICS TEACHING

Kant offers us an answer to the question of how knowledge is possible, leading us to reflections on the constitutive role of the world by the transcendental subject. In this direction, he presents us with the subject *Homo sapiens* as possessing the conditions of possibility of experience, that is, he proposes that knowledge is possible because the subject has intrinsic faculties that make it possible. Thus, Kant starts to investigate reason and its limits, instead of how the world must be in order to know it.

The subject in Kantian philosophy intrinsically presents two main sources of knowledge: *sensibility*, through which objects are thought of in intuition; and *understanding*, through which objects are thought of in concepts (Kant, 1998). The concept of object for Kant is not necessarily a literal object, a physical object, but it can be something broader, more general, an abstract object.

For Kant, the human being has a multiplicity of sensations of objects in the world, such as color, taste, smell, heat, texture, etc. These sensations are what we can call the Transcendental Aesthetics, also called the matter of the phenomenon, or the content of the experience. However, for all these impressions to have some meaning and enter the field of the knowable (of what can be known), it is necessary, in the first place, to be placed in their pure forms, in a priori forms of intuition, which for Kant are the space and time.

He proposes that pure forms of intuition appear before any mental representation of the object: before the word "flower" can be thought, the flower must be presented, received in the a priori form of space and time, allowing the experiencing of sensations such as colors, textures, smell, shapes and eventually patterns. This is the first step towards getting to know something, experiencing its most fundamental aspects. This Kantian way of thinking about how knowledge is possible converges with the reflection presented by Freire (2017) in his book *The Importance of the Act of Reading*, where Freire proposes that: "*Reading the world precedes reading the word".* Freire proposes a causal relationship, in which, in this pedagogical conception, experience precedes the more formal aspects and the more technically abstract language (Freire, 2017, 2019, 2020; Freire and Macedo, 2021). This conception agrees perfectly with the very definition of what Physics is:

> *Physics is a term originating from the Greek "physis" meaning "nature". It is the science that studies the laws that govern natural phenomena susceptible to being examined by observation and experimentation, seeking to frame them in logical schemes, in order to understand Nature in its most fundamental aspects.* (Silva; Medeiros, 2014, p. 14)

Physics is more than a branch of the natural sciences, it is a *fundamental science* (Hewitt, 2011).

The understanding (reading the word) as an effect to the cause sensitivity (reading the world) can stimulate, from a priori concepts, cognitions that exceed the limits of the very physical nature of the observable, leading the subject to the field of ideas, knowledge, in particular, scientific knowledge promoted by the scientific method. The scientific method can stimulate observation, experimentation, abstraction, induction, understanding of physical laws and theories, as well as their validity domains (Nussenzveig, 2002). This would be the transcendent subject, the one that goes beyond common limits and is elevated to creativity, breaking with methodism, and experiencing the nature of things. Reading the word, then, is nothing more than a sophistication of reading the world. Thus, language approaches orality and the subject become capable of dialoguing with the world, which Conceição Evaristo calls writing-experiences (Evaristo, 2021).

What Conceição Evaristo alerts us to is that one can only appreciate, if, one understands, and it is only in understanding (in biting the word), that the subject appropriates knowledge to see the heart of things. Furthermore, Evaristo observes with his poem "On Calm and Silence" that "there are submerged worlds, that only silence from poetry penetrates" (Evaristo, 2021). Again, we can weave a possible rapprochement with science, since it presents appreciable scenarios in the world of ideas. This leads us to a Transcendental Logic of knowledge that concerns understanding, the organization of thought that leads us to concepts, where the appreciation of the word occurs through its understanding. In a quote on the Critique of Pure Reason, Kant (1998) says:

> *Now logic in turn can be undertaken with two different aims, either as the logic of the general or of the particular use of the understanding. The former contains the absolutely necessary rules of thinking, without which no use of the understanding takes place, and it therefore concerns these rules without regard to the difference of the objects to which it may be directed. The logic of the particular use of the understanding contains the rules for correctly thinking about a certain kind of objects.*

What Kant is telling us is that Transcendental Logic is the very ability to organize our thoughts to understand the world, generating ways of thinking and reasoning. Through these cognitive forms, we move from a Pure General Logic to a Transcendental Logic.

Our thoughts commonly produce a science with its own rules, based on a General Logic. Pure General Logic is the one in which thinking is independent of intuition, not establishing investigative links on the object, considering only a priori concepts. General Logic is the common sense of our organization of non-reflective thought, not requiring the process of Transcendental Aesthetics in which, based on sensations, we are led to intuit and organize our thoughts. In those processes where there is no intuition, thoughts are organized in a simpler way, and that is why, in Pure Logic, the origin of objects is not discussed. For Pure General Logic, Kant (1998) tells us that:

1 - As general logic it abstracts from all contents of the cognition of the understanding and of the difference of its objects, and has to do with nothing but the mere form of thinking.

2 - As pure logic it has no empirical principles, a thus it draws nothing from psychology (as one has occasionally been persuaded), which therefore has no influence at all on the canon of the understanding. It is a proven doctrine, and everything in it must be completely a priori.

In short, Pure General Logic is concerned with the organization of thought, but not necessarily with knowing. Understanding its essence then, Kant invites us to move on to Transcendental Logic, where intuition is an important cognitive element in which the search for the origin of the object leads us to reason. Kant considers that in intuition there are searches for successively deeper knowledge, enabling an organizational process of reasoning that leads us to the elaboration and understanding of concepts, therefore, to knowledge. An important observation is that Transcendental Logic does not depend on empiricism or the induction of experimentation to exist, but does not rule it out. For Transcendental Logic, knowledge only needs intuition, since space and time already offer you the conditions for knowledge. For Kant, as space and time remain, then, they are sufficient to, through intuition, allow the elaboration of thoughts that take us from General Logic to Transcendental, creating the conditions for knowledge, and from knowledge, to generate concepts.

We can establish a relationship with the case of Newton's apple. We don't know if the story/anecdote of the apple falling is true or not (Martins, 2006), but we can imagine that if you are living and suddenly you observe the fall of a body, be it an apple or a simple leaf from a tree, not having induced her to fall, not having induced her to experience it, one can have an intuition about the fall from the process. From intuition, one can seek to know the origin of everything that happened, and based on the acquired knowledge, generate concepts. For Kant (1998), this process would be the passage from General Logic to Transcendental Logic:

In a transcendental logic we isolate the understanding (as we did above with sensibility in the transcendental aesthetic), and elevate from our cognition merely the part of our thought that has its origin solely in the understanding. The use of this pure cognition, however, depends on this as its condition: that objects are given to us in intuition, to which it can be applied. For without intuition all of our cognition would lack objects, and therefore remain completely empty.

While in General Logic the focus is on thought, in Transcendental Logic, the focus is on understanding things, on the relationships between understandings. Note that Kant notes in parentheses "(as we did above with sensibility in the transcendental aesthetic)", emphasizing that knowledge is promoted through sensibility. An implication of this observation, reiterated by Kant, is that Logic and Transcendental Aesthetics intersect, communicate. Transcendental Aesthetics indicates how we acquire knowledge through empiricism and/or induction, while Transcendental Logic indicates how we organize them. In general, Pure Logic is a simple logic, based on pure and simple human ability to think, while Transcendental Logic is the ability to think about how we organize these thoughts, seeking knowledge about objects that lead to the generation of concepts.

Kant still offers us a synthesis between rationalism and empiricism saying that in the absence of experience content, promoted only by intuition, thoughts are empty (rationalism). On the other hand, without concepts, they have no meaning (empiricism). In Kant's own words (Kant, 1998):

Without sensibility no object would be given to us, and without understanding none would be thought. Thoughts without content are empty, intuitions without concepts are blind.

Based on Kantian Transcendental Idealism, we worked on an initially real experiment, considering a simple logic, to then work on a mental experiment, in which with a simple sheet of paper, we discussed the concepts of distance and orders of metric magnitudes, taking a guided tour of the Universe.

III. THE UNIVERSE ON A SHEET OF PAPER

Considering that blank sheets bother designers, writers, poets (and physicists!), let's take a common A4* sheet of paper, as shown in Figure 1.

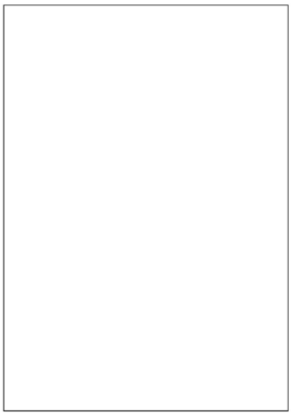

FIGURE 1. Sheet of A4 paper.

The thickness of this sheet is 0.1 mm, which can be obtained with a micrometer, or simply by using a ruler, measuring the thickness of a large set of sheets and then dividing it by the total number of sheets contained in the set. Here, we are not concerned, nor is it necessary for what we intend, the accuracy of the error associated with the measurements, importing in this discussion, only the conceptual ideas illustrated by a mental experiment. (Loos, 2015; Thenório, 2021) Folding the sheet in half and repeating this act over and over again, as depicted in Figure 2, it will soon become clear that the maximum number of folds possible for most people is only 6 or 7. Give it a try!

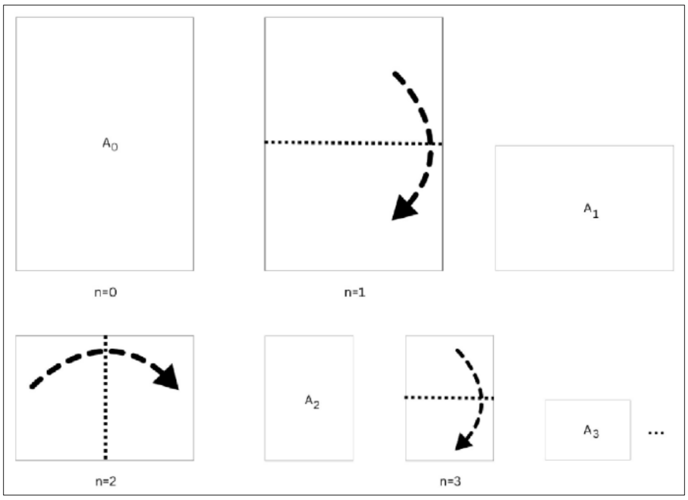

FIGURE 2. Sheet of A4 paper with successive folds.

Being A_0 the area of the sheet completely open (without any folding), its volume is then, $0.1 \cdot A_0$. Indicating the number of folds as an integer variable n, n = 0 being the absence of any fold, when we perform the first fold (to the middle of the sheet), n = 1, we note that while the area of the new surface, which is now A_1, reduces to half of the original one, $A_1 = \frac{1}{2} A_0$, the thickness of the ply sheet, 0.1 + 0.1 = 0.2 = 0.1 · 2 mm. We will indicate by L the thickness of the sheet, that is, in the case of a single fold, L = 0.1 · 2. Repeating the described process, when we make a second fold, n = 2, the new area, A_2, reduces to 1/4 of the original area, while the new thickness will correspond to 4 times the thickness of the open sheet, $L = 0.1 \cdot 4 = 0.1 \cdot 2^2 = 0.4$ mm. Folding the sheet a third time, n = 3, we will obtain as area, $A_3 = \frac{1}{8} A_0$, and consequently, the new thickness will be 8 times the thickness of the sheet original, that is, $L = 0.1 \cdot 8 = 0.1 \cdot 2^3 = 0.8$ mm. It is easy to see, therefore, that while two dimensions are reduced (leaf surface area), $A_n = \frac{1}{2^n} A_0$, a third dimension is increased (thickness or height of the folded assembly).

From the results, *primo ictu oculi* can be seen to be a characteristic behavior of a geometric progression to shape,

$$L = 0.1 \cdot 2^{n}. \tag{1}$$

Note that when there is no folding, if n = 0, we have the sheet's initial thickness, 0.1 mm. Figure 3 represents the thickness (or height of the sheet) initially without any folds and then with successive folds always in the middle of the sheet.

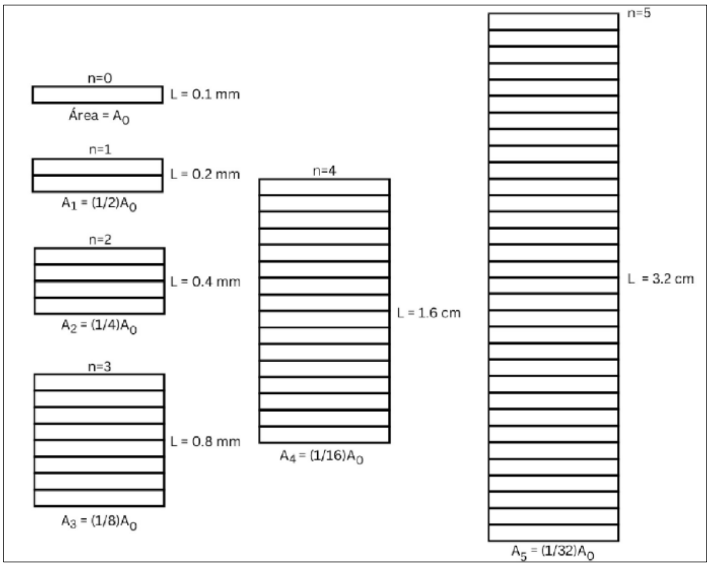

FIGURE 3. Thickness of the sheet of A4 paper with successive folds.

With 7 folds, n = 7, $L = 0.1 \cdot 2^7 = 12.8$ mm = 1.28 cm. This is the experimental limit of folds that some people can reach using a regular A4 sheet. Most don't make it past the 6th fold (Loos, 2015; Thenório, 2021)! From here, to continue folding, it will be necessary to transcend the experimental limit, go beyond the sensorial world and enter a scenario of abstractions, the world of ideas. So, in this new scenario, starting from expression 1, folding the sheet just 14 times, n = 14, we have $L = 0.1 \cdot 2^{14}$ = 1,638.4 mm ≈ 1.64m, which is approximately the average height of a human being. That is, the thickness of the folded set would be 1.64 meters! Doubling just 17 times, n = 17, $L = 0.1 \cdot 2^{17}$ = 13,107.2 mm ≈ 13 m, which corresponds to the approximate height of a 3-story building!

With 27 folds, we have a thickness of 13,421.77 meters, which is greater than the height of Mount Everest, in Nepal, whose summit is 8,848.86 meters above sea level, being, taking this level as a reference, the highest point on planet Earth.

With just 33 folds, the new thickness is 859 km, which is double the average height at which the International Space Station (ISS) is above the Earth's surface.

Carrying out the small number of 36 folds, the astonishing thickness of 6,871 km is reached, which is approximately 500 km greater than the average radius of the Earth, which is 6,378 km. The thickness of the sheet obtained with 36 folds is still approximately twice the road distance between the cities of Rio Branco in the state of Acre and the city of São Paulo. It is even greater than the geodesic distance (after all, the Earth is a geoid, not flat!) from Ponta do Seixas in the city of João Pessoa, in Paraíba state, to the source of the Moa River in the city of Mâncio Lima, in the state of Acre (extremes east-west of Brazil), which corresponds to 4,326.63 km. Tracing a geodesic arc, now starting from the north-south extremes of Brazil, connecting the source of the Ailã River in the city of Uiramutã, in Roraima, to the Arroio do Chuí River, in the city of Chuí, in Rio Grande do Sul, we have the distance of 4,378.41 km, that is, with 36 folds the thickness obtained would be even greater than that distance!

With 37 folds, we obtain a value of 13,743.9 km. This value is greater than that adopted by the International Astronomical Union (IAU) and by the International Union of Geodesy and Geophysics (IUGG), for the Earth's equatorial diameter, which is 12,756.28 km. The polar diameter is 12,713.5 km. Note that the polar diameter differs from the equatorial one by only 0.335%! This difference is so small that when we look at Earth from the Moon or even from the ISS, we will see it as a perfect sphere.

With 42 folds, the length is 439,804.65 km, greater than the 384,400.00 km corresponding to the Earth-Moon distance. Already with 50 folds, the value is 112.59 million km, a value close to the 149.6 million km corresponding to the Earth-Sun distance, which we define in Astronomy as 1 astronomical unit (AU). In Figure 4, we see an illustration that represents the length of the wire from 42 folds.

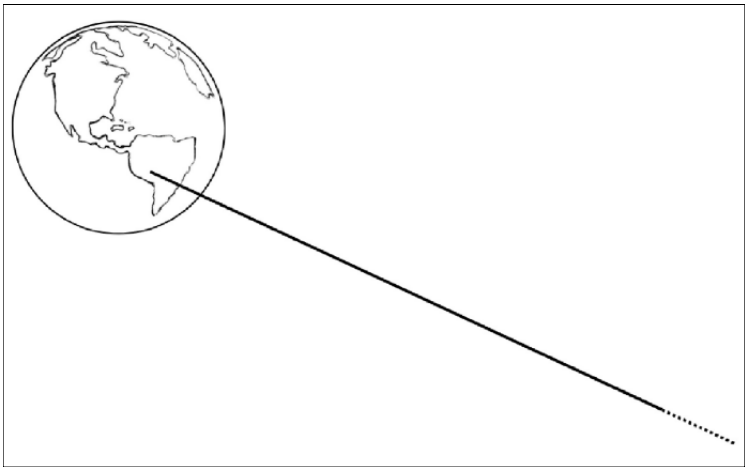

FIGURE 4. Wire length from 42 folds.

With 55 folds we pass by Neptune, and with 56 folds, obtaining the value of 7.2 billion km, we approach Pluto, a dwarf planet member of the Kuiper belt, discovered in 1930 and that on August 24, 2006, in a historic decision, science textbooks needed to be rewritten. On this date, Pluto, until then the 9th planet in the Solar System, was reclassified, starting to be considered a dwarf planet. Pluto is 7.38 billion km from Earth (aphelion), which is equivalent to 49.31 AU. That is, Pluto is 49 times farther from Earth than the Sun!

With 54 folds, the thickness of the sheet is 1.8 billion km. This is equivalent to 12 times the Earth-Sun distance or 12 AU.

We arrive at the 58-fold scenario. Here, with a thickness of 28.9 billion km, which is equivalent to 193 times the Earth-Sun distance, we have just passed the most distant object from Earth ever produced by mankind, the Voyager 1 space probe, launched into space on 5 September 1977, and which is in the year 2023, 23.3 billion km from Earth. On August 25, 2012, Voyager 1 reached 121 AU away, thus flying beyond the Heliopause, a region around the Sun where the solar wind is static, thus entering interstellar space (NASA, n.d.-a).

Folding the sheet a little more, with 66 folds, we arrive at the unbelievable mark of $7.38 \cdot 10^{12}$ km, which is approximately the distance that light travels in 1 year! As we know, 1 light year equals $9{,}461 \cdot 10^{12}$ km. With 68 folds, we get the value of $2.95 \cdot 10^{13}$ km, or 3.12 light-years, approaching the star Proxima Centauri (Alpha Centauri C), a red dwarf that orbits the Alpha binary system Centauri A and B, forming the Alpha Centauri triple system, and which is the second closest star to Earth, the first is the Sun, of course, and is 4,246 light years from Earth, or approximately 268,521.61 astronomical units, or $4.02 \cdot 10^{13}$ km, or 4.02 trillion km. This is so far away that on a scale where the Sun is only 1 meter away from Earth, Proxima Centauri would be 268.7 km away from Earth! While light from the Sun takes approximately 8 minutes from its surface to Earth, light from Proxima Centauri takes 4 years to get here! When we observe Proxima Centauri, we are therefore seeing it as it was 4 years ago. Do you want a time machine to see the past? Look at the stars in the night sky!

With 77 folds, we get a value of $1.51 \cdot 10^{16}$ km, which is equivalent to 100.94 million AU, or 1,596.07 light-years. Thus, when we performed the 77th folding, we arrived relatively close to the Ring Nebula, also called M57 or NGC 6720, one of the most remarkable planetary nebulae that is 2,300 light years away from Earth.

With 81 folds, we obtain the value of $2.42 \cdot 10^{17}$ km, or 25.58 thousand light years. At this point, we are very close to Sagittarius A* (read: Sagittarius A Star), a supermassive black hole with 4.1 million solar masses ($4.1 \cdot 10^{6}$ $M_\odot$) which is located in the center of the Milky Way at a distance of 26,000 light-years from Earth.

With 83 folds we obtain a value of 102,226.85 light-years, crossing almost the entire Milky Way, our galactic address, whose diameter is 105,700 light-years.

With 87 folds, the thickness of the sheet is $1.55 \cdot 10^{19}$ km, equivalent to 1.64 million light-years. At that point, we are 1/3 away from the Andromeda Galaxy (Messier 31, M31, NGC 224), which is 2.54 million light-years from Earth and which is the most distant object that humans can see with the naked eye. On some rare occasions, in extremely dark remote regions and with excellent meteorological conditions, it is possible to see a galaxy that is 3 million light-years from Earth, the Triangle Galaxy, being in this rare combination of variables, the most distant galaxy ever seen with the naked eye. As the conditions for this to occur are very rare, and as the Andromeda Galaxy can be more easily seen, Andromeda receives the title of being the most distant object seen with the naked eye by the human being.

With 88 folds we find the value of 3.27 million light-years thick. This value is equivalent to the diameter of the largest known galaxy, the supergiant IC 1101, which has a diameter of 3.9 million light-years and is at 1.05 billion light-years from Earth.

With 89 folds, the thickness is $6.18 \cdot 10^{19}$ km ≈ 6.18 quintillion km, or 6.5 million light-years. This thickness is greater than the radius of the Local Group, which is a group of more than 30 galaxies, including the Milky Way, that interact gravitationally and are scattered in a region of space around 10 million light-years in diameter (NASA, n.d.-b).

Continuing our folds, making the 93rd fold, we obtain the value of $9.9 \cdot 10^{20}$ km, which is equivalent to 10^5 million light-years. We can compare this value to the radius of the Virgo Supercluster, whose diameter is 200 million light years. This supercluster, also called the Local Supercluster, contains around 2,000 galaxies, including our own (NASA, n.d.-c). In a simplified comparison, considering the Earth our home, the Solar System would be our street, the Milky Way our block, the Local Group our neighborhood, and the Virgo Supercluster our city. There are other cities, there are forests and deserts. There are states, countries, unexplored seas, entire continents and who knows, other worlds.

At 95 folds, the value is $3.96 \cdot 10^{21}$ km (3.96 sextillion km), or 418.6 million light-years. This value is equivalent to only 1/4 of the distance through the Eridanus Supervoid, a region of low density of galaxies in which there is less matter than average. In these regions, as there is more matter distributed around the void than inside, at this observation scale, gravity pulls objects out of it. The Eridanus Supervoid region is illustrated in Figure 5.

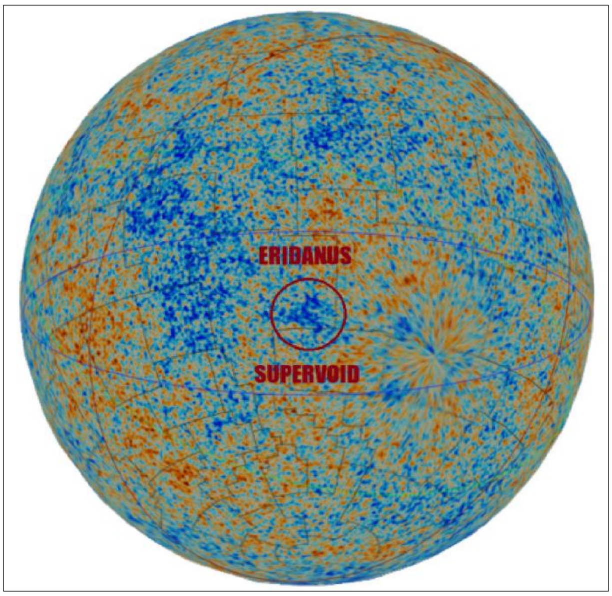

FIGURE 5. Location of Eridanus Supervoid in the celestial sphere. Source: (Wikimedia Commons, 2023).

We reached the 103rd fold. The scenario we are in now is cold and dark.

With 103 folds we obtain the inconceivable value of $1.01 \cdot 10^{24}$ km, which is the same as $1.07 \cdot 10^{11}$ light-years, or even 10^7 billion years-light. This is inconceivably larger than the diameter of the Observable Universe illustrated in Figure 6, which is approximately 93 billion light-years or $8.798479 \cdot 10^{23}$ km.

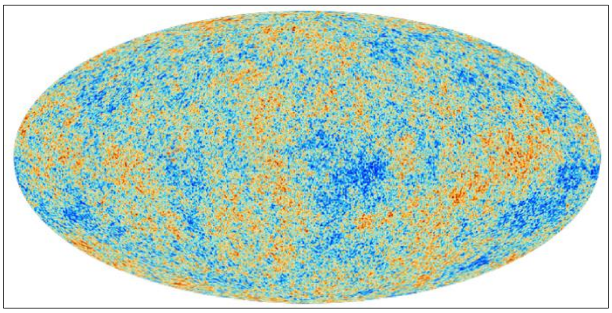

FIGURE 6. Map of the Observable Universe. Source: (NASA, 2013).

Where should we stop? What is the physical limit of this? The experimental limit we know, is the 6 or 7 folds we can do. But what about the theoretical limit? The theoretical physical limit for us to continue having a sheet of paper at each fold, or rather, for us to continue having its organic constituents, since it no longer makes sense to speak of a sheet, but of a long line of paper with a small diameter, is the limit the maintenance of its molecular structure, since if we undo this structure, all we will have will be just a set of atoms, decharacterizing the constituent identity of the paper sheet/thread object (Loos, 2015).

As a sheet of paper is essentially constituted by cellulose ($C_6H_{10}O_5$), which is formed by glucose monomers united by glycosidic bonds, being, therefore, a glucose polymer with a linear structure, the theoretical physical limit is that we have at the base of this entire pile of folded sheet, the thickness of a single molecule. We therefore have a one-dimensional string of $C_6H_{10}O_5$, as can be seen in Figure 7.

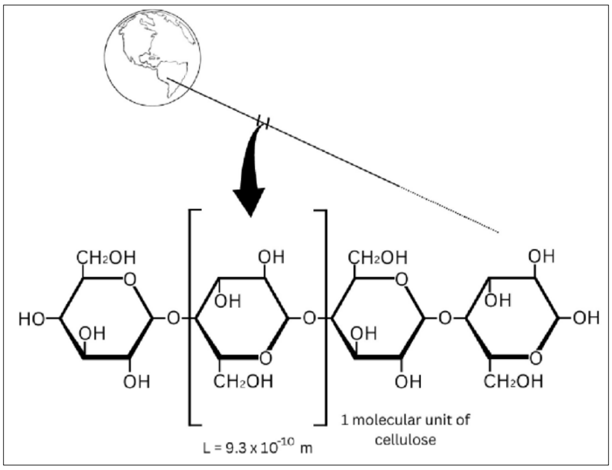

FIGURE 7. Cellulose molecular diameter string.

As in the cellulose molecule we have carbon atoms (atomic mass 12), hydrogen (atomic mass 1) and oxygen (atomic mass 16), in a mole of $C_6H_{10}O_5$, we will have 72 grams of carbon, 10 grams of hydrogen and 80 grams of oxygen, totaling 162 grams of molar mass. So, 1 mole of cellulose has 162 grams of molar mass. A sheet of plain A4 paper has a mass of approximately 5 grams, or 0.005 kg, and therefore contains 0.031 mol of cellulose. Since 1 mol of any substance has Avogadro's number of molecules, that is, $6.02 \cdot 10^{23}$ molecules, 0.031 mol of cellulose will have $1.87 \cdot 10^{22}$ molecules. The thickness of a cellulose molecule is 0.26 nm, while the length L is 0.93 nm (Saitoh, Ohno and Matsuo, 2013). So, being the length of 1 cellulose molecule 0.93 nm, and having an A4 sheet of paper, $1.87 \cdot 10^{22}$ molecules of $C_6H_{10}O_5$, we will have a cellulose thread of $1.7391 \cdot 10^{16}$ mm, or $1.7391 \cdot 10^{13}$ m, or $1.7391 \cdot 10^{10}$ km, or 17.4 billion km, or 0.0018 light years in length, or 0.00055 parsec, or 116.31 astronomical units. This corresponds to 116 times the Earth-Sun distance. Note that using Equation (1), we get $1.7391 \cdot 10^{16}$ mm $= 0.1 \cdot 2^n \Rightarrow 2^n = 1.7391 \cdot 10^{17} \Rightarrow \log_2 (2^n) = \log_2 (1.7391 \cdot 10^{17}) \Rightarrow n \approx 57$ folds. That is, with just 57 folds made on a sheet of ordinary A4 paper, we will have as a result a thread of just one cellulose molecule thick, whose length is 116 times greater than the Earth-Sun distance. We saw that on a scale where the Sun was at a distance of just 1 meter from Earth, Proxima Centauri would be on that same scale, 268.7 km from Earth. Continuing this metric analogy, with 57 folds, obtaining a thread a single cellulose molecule thick, the nth molecule of the thread would be only 116.3 meters away from Earth. As much as it is absurdly intriguing how much the length of the thread grows, as we carry out the successive folds, the nth molecule of cellulose that is at the top of our thread, will still be very close to the Earth, when compared with scales astronomical.

What if we now disassemble the cellulose molecules and stack all the atoms contained in the A4 sheet of paper? Therefore, the thickness of this atomic wire is only 1 atom, as represented in Figure 8.

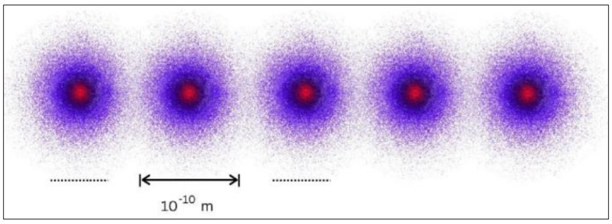

FIGURE 8. String with atomic diameter.

As we know, the average diameter of an atom is of the order of 1 Å = 10^{-10} m. This corresponds to a fraction equivalent to 10 billionths of 1 meter. In a cellulose molecule we have 21 atoms, so in a mole of cellulose ($6.02 \cdot 10^{23}$ molecules), we will have $1.26 \cdot 10^{25}$ atoms. As this amount of atoms corresponds to 1 mole of cellulose, which in turn corresponds to 162 grams, the 5 grams that make up the sheet will contain $3.89 \cdot 10^{23}$ atoms. Multiplying this number of atoms contained in the sheet of paper by the average diameter of an atom, 10^{-10} m, we get $10^{-10} \cdot 3.5 \cdot 10^{23} = 3.5 \cdot 10^{13}$ meters, that is, 35 billion km, which is approximately 235 times the distance from Earth to the Sun. Thus, using only 1 sheet of A4 paper, going to infinitesimals of the atomic-molecular level, and stacking up units of cellulose molecules or even atomic units, as expected, we are still unimaginably far from reaching the Observable Universe. How far? Very much! How much? As we have seen, with only 1 sheet of 5 grams, we are still “down the street from our house”, we haven’t even reached our closest stellar neighborhood.

What would be then, the area that a sheet of paper should have, so that repeating all the previously described mental experiment, we would have the length L of a thread, composed only by cellulose, of the order of magnitude of the Observable Universe? As we know, the Observable Universe has a diameter of $\approx 8.8 \cdot 10^{23}$ km. The dimensions of an A4 sheet of paper are 21.0 cm × 29.7 cm, which implies an area of 623.7 cm^2 or even $6.237 \cdot 10^{-8}$ km^2. For this area, corresponding to an A4 sheet of paper, if we fold it to the thickness of a single cellulose molecule, as we have seen, we will have a wire of length $1.7391 \cdot 10^{10}$ km. So, if in an area of $6.237 \cdot 10^{-8}$ km^2 we have, when we fold the sheet to the molecular dimension of cellulose, a length of $1.7391 \cdot 10^{10}$ km, using a simple rule of three, the area that a leaf will have to have to have a length of the order of magnitude of the diameter of the Observable Universe, $\approx 8.8 \cdot 10^{23}$ km, will be 3, 155.977.23 km^2. This area corresponds to more than 1/3 of the total area of the entire Brazilian territory, which is 8,515,767.049 km^2. In order to maintain the same proportion of the ratio between the edges x and y of an A4 sheet, we know that $\frac{x}{y} = \frac{21}{29.7} \Longrightarrow x = \frac{21}{29.7} y = 0.7070707071y$. Considering the area we calculated,

$x \cdot y$ = Area = 3, 155,977.23 km^2, so (0.7070707071y) y = 3,155,977.23 $km^2 \Rightarrow y = \sqrt{\frac{3{,}155{,}977.23 km^2}{0{,}7070707071}} = 2{,}112.688692 km.$

Like $\frac{x}{y} = 0{,}7070707071 \Longrightarrow x = 1{,}493.820287 km.$ Also, if an ordinary A4 sheet of area $6.237 \cdot 10^{-8}$ km^2 has 5 grams, which is equal to 0.005 kg, then a sheet that has an area of 3, 155.977.23 km^2 will have $2.53 \cdot 10^{11}$ kg of mass. Since in 0.005 kg there are $1.87 \cdot 10^{22}$ molecules, in $2.53 \cdot 10^{11}$ kg there will be $9.4622 \cdot 10^{35}$ molecules of cellulose. We saw that each cellulose molecule has a length of 0.93 nm = $9.3 \cdot 10^{-13}$ km, so, multiplying the number of molecules obtained by the length of a single molecule, we obtain the value of 8, $799846 \cdot 10^{23}$ km. That is, considering a sheet of paper with a thickness of 0.1 mm and approximate edges of 1,494 km and 2,113 km, if we fold the sheet until we obtain a thread that contains only a single molecule of cellulose thick, the length of this thread will be of same order of magnitude as the diameter of the Observable Universe!

IV. FINAL CONSIDERATIONS

Thought experiments and orders of magnitude are complementary tools that help us explore complex concepts and understand the magnitude of phenomena in different contexts. They allow us to imagine extreme or paradoxical situations, forcing us to confront our intuitions and previous concepts. The Universe on a Sheet of Paper thought experiment not only illustrates orders of magnitude and scientific notation in a mathematical way, but also leads us to question the very physical nature of observation and reality.

In Physics, order of magnitude is a tool for comparison that helps us put the scale of phenomena into perspective. When we talk about the number of molecules and atoms that make up a sheet of paper, compared to the size of the Universe itself, we can imagine and even “visualize” its vastness in relation to ourselves. This comparison leads us to reflect on our place in the Universe and the importance of understanding the different scales of existence. This activity, therefore, can create interdisciplinary connections of scale that range from Physics to Biology, from Mathematics to Sociology, from Geography to Literature.

The relationship between thought experiments and order of magnitude offers us a path to deepen our understanding of the world. While thought experiments provoke us to think critically about reality, order of magnitude provides us with a framework for understanding the magnitude and complexity of the phenomena around us. Together, these tools help us reflect on the deepest questions of science and philosophy, enriching our view of the world around us and the universe with just a sheet of paper.

For students, it is expected that activities of this nature can have positive implications for learning, so that the perception of the difference between scales of size, or even time and energy, allows them to contextualize information and make more meaningful comparisons, leading them to a better conceptual understanding. It is expected that activities that involve estimating and comparing stimulate critical thinking and develop in students the ability to evaluate and note the plausibility of everyday results and information. In addition, it is possible that teaching Physics, considering practical applications, contributes to a greater interest in Physics and Science in general, providing contexts for the development of analytical skills, proficiency and, therefore, greater depth.

Thus, based on Kantian Transcendental Idealism in Physics Teaching, we hope that the inclusion of activities such as The Universe on a Sheet of Paper will help to promote the enrichment of learning and can serve as another tool for socio-educational transformation based on the emancipation of critical thinking.

Although this work can be used in any Physics or Science class, its writing came about after we became aware of a rebellion that took place in a maximum-security prison in Rio Branco in the state of Acre (Portal G1, 2023), and reflections on how work on teaching Physics and Science for people deprived of liberty in prison systems, where the input of materials for the development of experimental classes by teachers may not be recommended. The experiment discussed in this paper uses a sheet of paper and imagination. "... there are submerged worlds, that only silence from poetry penetrates" (Evaristo, 2021).

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