MULTICRITERIA ANALYSIS WITH UNKNOWN PREFERENCES: AN APPLICATION OF THE SMAA-2 METHOD

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ABSTRACT

Applications of Multicriteria Decision Aiding are normally dependent on the preferences concerning alternatives for each criterion. They also depend on the measurements of the importance of criteria. Over the last decade, as a response to the fact that such preferences or measurements are often either not available or highly uncertain, Finnish researchers have developed a family of analytical methods called SMAA. Methods belonging to this family include SMAA-1, SMAA-D, SMAA-O, SMAA-2, SMAA-3, SMAA-A, SMAA-TRI, Ref-SMAA and SMAA-P. They consist, in essence, of formulating inverse problems in the weight space. These problems allow for the solving of multidimensional integrals and can be approached by the Monte Carlo simulation. In this article, the principal concepts of SMAA methods are presented. An example application to real data of one of the most important among these methods, the SMAA-2 method, is developed to demonstrate the main features of this approach. The article closes by addressing the appropriateness of using SMAA methods when the above-mentioned limitations prevail.

KEYWORDS: Multicriteria decision aid - Stochastic preferences - Modelling of uncertainty - Monte Carlo simulation

1. INTRODUCTION

Decision-making in the corporate world faces ever more complex challenges, influenced by diverse factors, such as the availability of financial and human resources; internal and external policies; and technical and strategic variables, among others. The speed at which decisions can be made and implemented can result in gains of the competitive advantage at some moments, as well as significant losses at others. This aspect is also affected by the lack of a priori knowledge of future scenarios, generating doubt regarding the results from decision-making in complex situations and an increasing need to facilitate the decision-making process in areas of risk and uncertainty.

Although the advent, in the second half of the last century, of a field of knowledge called Multicriteria Decision Aiding has considerably enriched the process of decision analysis in complex environments, decisions are always made in the midst of risk and uncertainty (BELTON & STEWART, 2002; GOMES & GOMES, 2012; SANT'ANNA, 2010).

On the other hand, although there is a significant array of analytical methods to aid multicriteria decision-making, almost all of these methods traditionally start from the premise that the preferences of the agents are revealed in a precise way or, alternatively, that these preferences can be treated by techniques for modelling imprecision and uncertainty, such as fuzzy sets, rough sets, interval methods or even the evidence theory of Dempster-Shafer (FIGUEIRA, GRECO & EHRGOTT, 2005; SALICONE, 2007). In this way, in an attempt to analyze and solve decision-making problems in the presence of multiple criteria, as well as of lacking information, imprecision and uncertainty, Finnish researchers have recently developed a new family of methods. This family is called Stochastic Multicriteria Acceptability Analysis (SMAA). The methods belonging to this family have been developed to deal with multicriteria selection problems (Pα), ordering or ranking (Pγ), and classification (Pβ).

Accepting the lack of knowledge of the parameters of the problem, the SMAA methods are based on an inverse analysis in the space of feasible values for these parameters. Thus, taking a ranking problem as an example, what a corresponding SMAA method (in this case SMAA-O) does, in essence, is calculate the probabilities for the prevailing values for the parameters, assigning to the alternatives determined positions in the desired ranking.

This article presents the central ideas of the SMAA methods, as well as an example of their application to real data for one of the most important of these methods, SMAA-2.

2. METHODS OF THE SMAA FAMILY

The underlying vision of the SMAA methods consists of approaching the decision-making problem in an inverse way. Instead of preliminarily seeking the values of the parameters and, through a calculation using a multicriteria method, obtaining a solution for the problem, they identify the values of the parameters of the problem that result in different solutions (TERVONEN & LAHDELMA, 2007; TERVONEN & FIGUEIRA, 2008).

The SMAA methods have their roots in the articles of Charnetski (1973) and Charnetski & Soland (1978) in that these authors seek to calculate, for each alternative, the volume (in a generalized sense) of the multidimensional space of weights that make each alternative the most preferred.

This aim is also related to the proposal of Rietveld (1980), which was afterwards expanded upon in Rietveld & Ouwersloot (1992). These authors present formulations similar to those of the first two authors, although for problems with ordinal criteria and ordinal information from the preferences. Finally, it can also be said that the work of Bana e Costa (1986) on the overall compromise criterion is another precursor of the SMAA methods, to the extent that this author proposes, although only for three criteria, a calculation of the quantity of conflict between the preferences of different decision-makers in order to define a joint probability density function for the weight space.

The SMAA family methods are, in addition to SMAA-1 (LAHDELMA, HOKKANEN & SALMINEN, 1998), SMAA-D (LAHDELMA, SALMINEN & HOKKANEN, 1999; 2002), SMAA-O (LAHDELMA, MIETTINEN & SALMINEN, 2003), SMAA-2 (LAHDELMA; SALMINEN, 2001), SMAA-3 (LAHDELMA & SALMINEN, 2002), SMAA-A (LAHDELMA, MIETTINEN & SALMINEN, 2002), Ref-SMAA (LAHDELMA, MIETTINEN & SALMINEN, 2005), SMAA-TRI (TERVONEN et al., 2007), and SMAA-P (LAHDELMA & SALMINEN, 2009).

Let there be m alternatives X = $\{x_1,..., x_i, ..., x_m\}$ and n criteria $\{g_1,...,g_j,...,g_n\}$. The character value g_j (x_i) designates the evaluation of the alternative x_i according to the criterion g_j. The family of SMAA methods is based on the premise that the preference structure of the decision-maker can be represented by a utility function (KEENEY & RAIFFA, 1976) of real value $u(x_i,w)$ that is associated with the different alternatives and dependent upon the values of the components of the weights vector $w \in W$. This family also considers that there may be multiple decision makers and that each one of these decision makers has a preference structure represented by an individual weights vector w and a utility function of real value $u(x_i,w)$, with a consensually accepted form. The linear utility function, shown in (1), is the most commonly used as follows:

$$u(x_i, w) = \sum_{j=1}^{n} w_j u_j(x_{ij}) \tag{1}$$

With the abovementioned criteria weights having non-negative and normalized values, the feasible weight space W is given by (2).

$$W = \{w \in R^n : w \geq 0 \wedge \sum_{j=1}^{n} w_j = 1\} \tag{2}$$

The total utility function shown in (1) is a convex function of the partial utility functions $u_j(x_{ij})$, reading the values of the weights of the criteria on a scale from 0 to 1. If the values of the weights of the criteria were precisely known and a single vector of weights, determined, the problem would be solved easily by the calculation of the value of the utility function for each alternative, followed by the selection of the alternative with the greatest utility.

Nevertheless, as SMAA methods were developed precisely for situations in which the values of the weights are not generally known, these values are here considered as stochastic variables ξ_{ij} corresponding to the deterministic evaluations $g_j(x_i)$ with an assumed or estimated joint probability distribution and joint density function $f_\chi(\xi)$ in the space $\chi \subseteq R^{mxn}$. Thus, the unknown or partially known preferences of the decision-makers can be represented by means of a distribution with joint density function $f_W(w)$ in the feasible space of weights W.

The uniform distribution of the weights in W is used when there is no information on the weights. In this case, the density in the space of weights is given according to (3):

$$f_w(w) = 1/vol(W) \tag{3}$$

where *vol* refers to the volume of a set in a multidimensional space.

Most frequently, the values of the criteria can be treated as independent stochastic variables with the joint density function f being represented by a product of density functions f_{ij}, as in (4):

$$f(\xi) = \prod_{ij}^{mn} f_{ij}(\xi_{ij}) \tag{4}$$

The utility function is then employed to map the stochastic criteria, as well as the weight distributions, into utility distributions $u(\xi_i, w)$. The SMAA methods thus permit the set of favorable weights to be determined, for any alternative x_i, by $W_i(\xi)$, defined by (5):

$$W_i(\xi) = \{w \in W : u(\xi_i, w) \geq u(\xi_k, w, k = 1,..., m)\} \tag{5}$$

Any weight $w \in W_i(\xi)$ makes the general utility of x_i greater than or equal to the utility of any other alternative. All of the later analyses are based on the properties of these sets of weights. By means of Monte Carlo simulation, a set of indices (in other words, of indirect measurements of preference for the alternatives compared) of the SMAA methods is determined. When the number of replications in the simulation is sufficiently large, the margins of error are negligible. Among the possible indices, the acceptability index describes the contribution of different parameters' evaluations in making a given alternative the most preferred. The acceptability index a_i is computed for an alternative as the expected multidimensional volume of the set of favorable weights. This computation is a multidimensional integral over the criteria distributions and over the favorable weight space, as shown in (6):

$$a_i = \int_x f_X(\xi) \int_{W_i(\xi)} f_W(w) dw d\xi \quad (6)$$

The acceptability index can be used to classify the alternatives as stochastically efficient (a_i » 0) or inefficient (a_i equal to 0 or close to 0 – a ceiling of, for instance, 0.05 may be set). An acceptability index equal to 0 points that the alternative could never be considered the best with the preference model adopted, i. e., with the assumed utility function. On the other hand, for stochastically efficient alternatives, the acceptability index evaluates the efficiency, considering the lack of knowledge and the uncertainty about the measurements of the criteria and of the preferences of the decision makers.

In addition, the central weight vector w^c_i is defined as the expected center of gravity of the favorable weight space. Thus, the central weight vector is computed as a multidimensional integral on the distributions of the weights for the criteria, according to (7):

$$w_i^c = \int_x f_X(\xi) \int_{W_i(\xi)} w f_W(w) dw d\xi \Big/ a_i \quad (7)$$

For the presumed distribution of the weights, the central weight vector represents the preferences of a typical decision maker, supporting the alternative x_i with the adopted preference model. The central weights can be presented to the decision-maker to improve his understanding of how different weights can lead to different choices. The central weights vector is naturally undefined for inefficient alternatives; for these, it is stipulated $w^c_i = 0$.

Other useful measurements are the confidence factors, which express the probability that each given alternative reaches the first position when its central weight vector is chosen. Confidence factors can be calculated for any given central weight vectors. What is in fact measured by these factors is the accuracy of the measurements of the criteria by which the efficient alternatives are identified (TERVONEN, LAHDELMA & SALMINEN, 2004). The confidence factor p_i^c is the probability of an alternative obtaining the first rank when its central weight vector is chosen and its evaluations are taken as the reference point. The confidence factor is computed as a multidimensional integral over the criteria, and the distribution of preferences, using (8):

$$p_i^c = (\int_x f_x(\xi) \int_{v_i^l(X)} \delta^n (W - W_i^c)\ \delta^n (X_R - X_i^c) * f_s(S)\ dW\ dX_R\ ds\ dx) \quad (8)$$

In (8), the Dirac function δ is used to restrict the distribution of weights of the criteria to the central weight vector and the distribution of the points of reference to the central reference point.

The Dirac function may also be used to deal with deterministic criteria. In the formulations presented above, if the *j*-th criterion is to be treated as determined, with exact evaluations x_{ij}, the ξ_{ij} can enter as a special case of independent stochastic variables with density functions, as shown in (9):

$$f_{ij}(\xi_{ij}) = \delta(\xi_{ij} - x_{ij}) \tag{9}$$

For the computation of the acceptability indices and central weight vectors, (6) and (7) are reduced to (10) and (11):

$$a_i = \int_{w_i} f_W(w)dw \tag{10}$$

$$w_i^c = \int_{w_i} \frac{wf_W(w)dw}{a_i} \tag{11}$$

In practice, the multidimensional integrals of Equations (6) and (7) are calculated through the Monte Carlo simulation, where sampling problems are randomly generated by criteria matrices and weight distribution vectors.

3. PECULIARITIES OF THE SMAA-2 METHOD

The SMAA-2 method (LAHDELMA & SALMINEN, 2001) was developed for decision problems which involve, in most cases, evaluations of alternatives and estimates of criteria weights that are represented by stochastic variables and involve multiple decision makers. This method is especially recommended for situations where neither all of the criteria nor the weights are precisely known. SMAA-2 can be considered an emblematic method of the SMAA family because while most of the methods of the family developed later were designed to address specific problems, it generalizes SMAA-1 by considering the possible interest on secondary ranks. This method contains the great majority of indices used in the other methods of this family. JSMAA (TERVONEN, 2010; 2012), the most recent software for the SMAA approach, implements two methods: SMAA-TRI for ELECTRE-TRI (Roy, 1996) and SMAA-2 for models based on the multi-attribute value theories.

The SMAA-2 method uses, similar to the other SMAA methods, an inverse analysis of the weight space to describe, for each alternative, the type of preference that makes it the most preferred or places it in a determined position in a pre-defined ranking. The decision problem is represented by a set of m alternatives that will be evaluated by n criteria. The preference structure of the decision-makers is represented by a value function $u(x_i, w)$.

This value function maps the different alternatives x_i and weight vectors w to a value that quantifies the individual preferences of each of the decision makers. Uncertain or imprecise criteria are represented by stochastic variables ξ_{ij} together with the density function $f_X(\xi)$, where $X \subseteq R^{mxn}$. The unknown or partially known preferences are represented by a weight distribution function with a joint density function $f_w(w)$ where W represents the space of allowed weights. If there is a total lack of information regarding preference, the function is represented by a uniform distribution for the weights in W, $f_w(w)$={1/(vol(W)}. The weight space is defined according to the nature of the problem, even though, as a rule, the weights are positive and normalized. W, therefore, will be described by the restrictions in (2) above.

The utility function is used to transform the stochastic criteria and weight distributions into utility distributions $u(\xi_i,w)$. Based on the utility distribution, the classification of each alternative is defined as an integer that varies from *1*, for the best classification, to m, for the worst classification. This value is determined by means of the classification function presented in (12):

$$rank(i,\xi,w) = 1 + \sum_{k=1}^{m} \rho(u(\xi_k, w) > u(\xi_k, w)) \tag{12}$$

in which ρ (true) = 1 and ρ (false) = 0. Based on the analysis of the sets of stochastic weights, the chance of classification as r-th is measured by the volume of the favorable set of weights W_i^r, given in (13):

$$W_i^r(\xi) = \{w \in W : rank(i,\xi,w) = r\} \tag{13}$$

Any vector of weights w belonging to $W_i^r(\xi)$ assigns utilities for the alternatives, such that the i-*th* generic alternative x_i has the position or rank r_i.

The most important descriptive measurement of SMAA-2 is the acceptability index, computed as shown in the previous section. In SMAA-2, this index is accompanied by indices that measure the multiplicity of different preferences for weights that grant alternative x_i each different rank. The computation along all the evaluated weights makes the acceptable option feasible for each determined classification. The acceptability index b_i^r is calculated, extending (6) by means of the multidimensional integral in (14):

$$b_i^r = \int_{\xi \in X} fx(\xi) \int_{w \in W_i^r(\xi)} fw(w)\ dw\ d\xi \tag{14}$$

The most highly indicated alternatives are those with high values for the acceptance index of reaching the best classifications.

The acceptance indices vary inside the interval [0, 1]. The value 0 indicates that the alternative will never obtain the considered classification in the ranking, while 1 indicates that the alternative will always obtain that classification, independent of the choice of weights.

4. AN APPLICATION OF THE SMAA-2 METHOD

SMAA-2 allows for two levels of statistical modeling, including not only a statistical model for the space of the weights for the criteria, but also a statistical distribution for the evaluation of each alternative according to each criterion. This makes the exact computation of the integrals that produce the acceptability indices, central weights vectors and confidence factors for each alternative extremely cumbersome. Difficulty notwithstanding, employing the simulation techniques of the jmaf software makes it possible to solve even considerably large problems.

We consider now a problem with 23 alternatives. The data set, formed by evaluations under four criteria, includes 23 stores of a clothing retail sales network. The four criteria are, by one side, two production factors: area of the store and the dimension of the staff of salespeople, and by the other side, two indices of results of the production: sales value and number of sale operations.

This problem has been studied before by Ribeiro *et al.* (2010), who applied Data Envelopment Analysis to compare the stores from a productivity point of view, taking as DEA inputs the area and staff and, as DEA outputs, the value of sales and number of sales. The measurements of staff dimensions, sales value and number of sales vary with time and are given in the data set collected by Ribeiro et al. (2010) shown in Table 1 as monthly averages over a 12-month period. Then, while the area's values are considered exact, the values of these averages are treated as the means of normal distributions for the other variables. A common standard deviation is assumed for the distributions of the measurements of each of the 4 criteria and estimated by the sample standard deviation of the set of 23 measurement averages for the criterion.

Summarizing the analysis of Ribeiro *et al.* (2010), we were able to obtain the best productivity scores for Store 3, regardless of whether we considered the measurements as exact or applied a stochastic DEA (KAO & LIU, 2009).

SMAA-2 is applied using the software JSMAA on the data set of 23 alternatives, evaluated according to 4 criteria. For the 1^{st} criterion, with a negative orientation, the alternatives are evaluated by the values of the area column in Table 1, identified as exact values. For the second, also with a negative orientation, the entries are normal distributions with means at the values of the sales people column of Table 1 and standard deviation 4, the sample standard deviation of that column vector.

The entries for the third and fourth criterion, with positive orientation, are normal distributions with means at the observed values in the respective columns of value of sales and number of sales and standard deviations of 85,000 and 14,000, respectively, analogously estimated by the sample standard deviations.

With 23 alternatives, the data set is too large for the graphical display resources of JSMAA, but precise numerical results were obtained. Table 2 presents the four main results for the 23 alternatives: the two main acceptability indices, i. e., the probabilities of being ranked first and second, the confidence factors and the center vectors of weights.

As expected, the acceptability indices added approximately to 1. Store 3 presents, clearly, the highest probability of being ranked first, followed by Stores 12 and 11. Store 12 presents the highest probability of being ranked second, followed, closely, by Store 3 and, at a larger distance, by Store 11.

Store 3 also presents the highest confirmation factor for the analysis based on its central vector of weights. The central vectors of weights for Store 3 were as follows: 0.54 for the area, 0.24 for the number of salespeople, 0.15 for the value of sales and 0.07 for the number of transactions. These data are very similar to those of Stores 11 and 12; this makes it easier to conclude the analysis by choosing Store 3 as that one of highest productivity.

In fact, all 11 stores with a probability of at least 0.01 of being ranked first or second by SMAA-2 presented similar patterns of central weights. Their larger weights were for area, followed by salespeople and then by much smaller weights for the value and number of sales, in that order. The information provided by the method leads us to conclude that our decision regarding productivity is determined mainly by the efforts to limit the use of resources and less by the efforts to raise the output generated by the stores.

This suggests the development of a complementary analysis to the productivity analysis that is centered only on the production of outputs. This implies evaluating the performance of the stores only in terms of their ability to generate sales, independent of the volume of resources employed. This may be done by withdrawing from the data set the two first criteria and considering only the output evaluations. The results provided by SMAA-2 for this analysis are shown in Table 3. This table shows that, according to this new point of view, Store 20 presents the best performance, followed by Stores 16 and 15, and Store 3 appears only in the fourth place. The analysis of the influence of the criteria shows a larger influence of value than of number of sales, which is a reasonable result, as the value of sales hints more precisely at the volume of output rather than the number of transactions.

5. CONCLUSION

This article shows how SMAA methods can be used precisely to solve the inverse problem of traditional multicriteria analysis. Here, we try to identify the preferences of decision makers, which serve as input variables for the use of multicriteria methods. When these preferences are ignored, the SMAA methods estimate the likelihood associated with determined results by using a small set of measurements of a probabilistic nature. Thus, for example, in a problem of type Pβ, the SMAA-TRI method is used to estimate the likelihood that each alternative would be inserted into one of the pre-defined classes. In the example of the application presented in this article, using the SMAA-2 method and the measurements presented above, the likelihood of associating each of the alternatives with distinct positions in the ranking was estimated. In this way, it is possible to attain the best solution for the problem.

An interesting research development related to the SMAA family arose when decision-making agents were shown to behave consistently with Prospect Theory (TVERSKY & KAHNEMAN, 1991). The TODIM multicriteria method (GOMES & LIMA, 1992; GOMES & RANGEL, 2009) is one of the rare attempts that tried to make this theory operational. The path to this development was designated by the outlining of the SMAA-P method undertaken by LAHDELMA & SALMINEN (2009). Although both authors used linear difference functions in the formulation of SMAA-P, these functions are not linear in the TODIM method. In addition, in the context of a multicriteria decision problem with multiple decision-makers, the reference points should be expected to be affected by the lack of knowledge just as much as by the preferences of those decision-makers. Therefore, combining the SMAA family methodology and the TODIM method by making SMAA-P operational would constitute a worthwhile research objective to pursue.

Store	Area	Salespeople	Value of Sales	Number of Sales
1	61.6	15.08	278920.51	4630.5
2	48.8	13.42	157397.06	2907.83
3	42.2	16.67	312420.66	5536.58
4	68	11.33	129946.52	1722.75
5	63	11.83	109870.04	1925.92
6	79.7	11.08	126053.24	2139
7	54.3	12.92	139646.78	2443.5
8	82.3	13.25	132935.65	1788.67
9	54.1	10.71	102051.63	3040.71
10	78.7	12.67	173438.43	3121.67
11	46.3	11.5	116873.48	2580.42

12	43.8	12.83	162551.56	2844
13	84.2	15	151945.61	3014.5
14	125.2	21.75	258993.74	4851.58
15	58.9	12.83	105833.06	1502.17
16	82.2	19.5	328325.62	5807.5
17	65.9	20.42	345457.84	4998.75
18	58.2	12.58	199445.9	3163.08
19	72.5	12.92	142990.16	2526.42
20	100	25.17	395181.4	6973.17
21	70	13.42	228666.05	3659.83
22	80	18.83	210533.31	3360.83
23	60	14.5	215599.13	2551.58

Table 1 - Data of 23 Stores

Acceptability Indices				Central weights and Confirmation factor			
Store	**Rank 1**	**Rank 2**	**CF**	**Area**	**People**	**Value**	**Transactions**
1	0.01	0.02	0.02	0.39	0.32	0.21	0.08
2	0.06	0.11	0.08	0.47	0.30	0.16	0.07
3	0.44	0.22	0.5	0.54	0.24	0.15	0.07
4	0.00	0.00	0.00	0.4	0.34	0.19	0.08
5	0.00	0.01	0.01	0.41	0.35	0.15	0.08
6	0.00	0.00	1.00	-	-	-	-
7	0.02	0.04	0.03	0.44	0.34	0.16	0.06
8	0.00	0.00	1.00	-	-	-	-
9	0.04	0.05	0.06	0.45	0.34	0.15	0.07
10	0.00	0.00	0.00	0.35	0.34	0.29	0.02
11	0.15	0.19	0.19	0.51	0.30	0.14	0.06
12	0.25	0.29	0.29	0.53	0.27	0.13	0.06
13	0.00	0.00	1.00	-	-	-	-
14	0.00	0.00	1.00	-	-	-	-
15	0.00	0.01	0.01	0.43	0.36	0.15	0.07
16	0.00	0.00	1.00	-	-	-	-
17	0.00	0.00	0.00	0.36	0.29	0.24	0.12
18	0.02	0.03	0.03	0.41	0.32	0.19	0.08
19	0.00	0.00	0.00	0.34	0.30	0.24	0.11

20	0.00	0.00	1.00	-	-	-	-
21	0.00	0.00	0.00	0.39	0.33	0.22	0.07
22	0.00	0.00	1.00	-	-	-	-
23	0.01	0.02	0.02	0.40	0.30	0.20	0.101

Table 2 - Results of SMAA for Productivity Model

Acceptability Indices			Central weights and Confirmation factor		
Store	**Rank 1**	**Rank 2**	**CF**	**Value**	**Transactions**
1	0.04	0.08	0.1	0.04	0.78
2	0.00	0.00	0.00	0.00	0.00
3	0.09	0.17	0.17	0.10	0.77
4	0.00	0.00	0.00	0.00	0.90
5	0.00	0.00	0.00	1.00	0.00
6	0.00	0.00	0.00	0.00	0.93
7	0.00	0.00	0.00	0.00	0.93
8	0.00	0.00	0.00	0.00	0.96
9	0.00	0.00	0.00	1.00	0.00
10	0.00	0.00	0.01	0.00	0.85
11	0.00	0.00	0.00	0.00	0.57
12	0.00	0.00	0.01	0.00	0.74
13	0.00	0.00	0.01	0.00	0.80
14	0.02	0.06	0.09	0.03	0.78
15	0.00	0.00	0.00	1.00	0.00
16	0.14	0.21	0.18	0.14	0.75
17	0.15	0.21	0.19	0.15	0.78
18	0.00	0.01	0.02	0.00	0.84
19	0.00	0.00	0.00	0.00	0.87
20	0.53	0.21	0.11	0.54	0.73
21	0.01	0.02	0.04	0.01	0.81
22	0.00	0.01	0.03	0.00	0.82
23	0.00	0.01	0.02	0.01	0.85

Table 3 - Results of SMAA for Production Model

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